A Comparison of Cognitive Equations of Mathematics Learning Process between the American and Singaporean Students with Dyscalculia

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ABSTRACT

This study provided a comparison between the American and Singaporean students identified with dyscalculia, also known as mathematics learning disability, in their performance based on the administration of the Test of Mathematical Abilities-2nd Edition (TOMA-2). TOMA-2 has been normed on 2082 American students, aged between 8-0 through 18-11, representing 26 states between 1990 and 1992. Among the participants, 38 or 5% of all the participants were identified with dyscalculia. However, TOMA-2 did not provide much background information (e.g., age, gender and race) about these 38 American students for a proper comparison with the 40 Singaporean students, aged 9-6 through 10-11. Despite missing information about the American cohort, findings of this study suggested that there was an obvious difference in terms of the cognitive equations for difficulties in mathematics learning process between the American and Singaporean students with dyscalculia.

Keywords: Cognitive equation, Dyscalculia, Learning difficulty, Mathematics, TOMA-2

INTRODUCTION

In both primary and secondary schools, mathematics has always been a challenging academic subject for many students. It consists of numerous domains that continue to develop in a cumulative manner toward increasingly complex topics (Wendling& Mather, 2009). Hence, many students see mathematics as a boring and tedious subject that requires them to memorize rules and know how to apply them. Should they get their answers right for the exercises they did, it is often assumed that they have understood the mathematical concepts. However, this is not the case. In fact, learning difficulties, especially in the domain of mathematical comprehension (essential for solving mathematical problems), begin to crop up at a higher level revealing a serious lack of real understanding of fundamental mathematics concepts.

With its own set of vocabulary, jargons and symbols that convey meanings best understood within its own context, learning mathematics is like acquiring a new language. However, the semiotic system of mathematics is very different from that of the linguistic system that we called language. In learning to count and compute, the numerical knowledge awareness (or number sense) plays a similar role – like that of phonemic sense in reading – in mathematics learning (Chia & Kho, 2011; Wendling & Mather, 2009). The early numerical knowledge awareness usually develops during the preschool years and most young children have an initial understanding in place by the ages of 4 and 5 years (Griffin & Case, 1997).

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When children encounter difficulties in mathematics learning, the seemingly common reaction to resolve the issue is to get them to practice more because most of us believe that practice makes perfect. Chan (2009) has encouraged mathematics teachers to take time to reflect and ponder why there are children who continue to fail learning mathematics despite extra remedial lessons and provision of learning support for mathematics. This will allow teachers the opportunity to observe and/or examine the errors these children have committed, misconceived or responded in certain ways when working out sums or mathematical problems. From their observation or examination of error patterns, teachers can actually learn more and understand better and thus, become better equipped to manage the various learning difficulties children encounter in their mathematics learning (Chia & Kho, 2011).

Defining the Learning Disabilities in Mathematics

According to Wendlingand Mather (2009), an estimated 5-8% of school-age children manifest significant problems in mathematics learning, including those with dyscalculia (see Chia & Kho, 2011; Geary, 2004, for separate reviews), and more than 60% of them diagnosed with a learning disability in reading are also performing poorly in mathematics (McLeskey& Waldron, 1990). As a result, many researchers (e.g., Geary, 2004; Jordan &Hanich, 2000) have embarked on their respective investigations into the various cognitive factors affecting mathematics learning. The questions that pose challenges to our understanding of dyscalculia are: What exactly constitutes the many learning disabilities in mathematics? How do we define it? Is there a distinction between mathematics learning disabilities and difficulties, or can both be termed as dyscalculia?

According to the Australia's National Health and Medical Research Council (NHMRC) (1990), learning difficulties and learning disabilities are different. *Learning difficulties* is a generic term referring to "the substantial proposition (16-19%) of those who exhibit problems in developmental and academic skills ... are considered to result from one or more of the following factors: intellectual disability, physical and sensory defects, emotional difficulties, inadequate environmental experiences, lack of appropriate educational opportunities" (p.2). On the other hand, *learning disabilities* refers to "smaller proportion (2-4%) … who exhibit problems in developmental and academic skills significantly below expectation for their age and general ability ... include severe and prolonged directional confusion, sequencing and short-term retention difficulties … presumed to be intrinsic to the individual, but are not considered to be the direct result of intellectual disability, physical and sensory defects or emotional difficulties ... or derive directly from inadequate environmental experiences, or lack of appropriate educational experiences" (NHMRC, 1990, p.2). Both terms mean different learning problems in terms of the degree of severity as well as their respective prevalence.

Many other terms have been used to describe dyscalculia, such as mathematical disability (Chia & Kho, 2011), arithmetic learning disability (Geary & Hoard, 2001), number fact disorder (Temple & Sherwood, 2002), number blindness (Butterworth, 2003), and psychological difficulties in mathematics (Allardice & Ginsburg, 1983).

With a multiple number of terms used to describe dyscalculia, it goes to show that it is never easy to define exactly what dyscalculia is. There is still no one agreeable operating definition of dyscalculia among the researchers. According to Chia and Yang (2009), there are, perhaps, two approaches (depending on which one most prefer to use) to defining dyscalculia. The first approach proposed by Macaruso et al. (1992) is to examine dyscalculia by identifying the three key areas of difficulties: number processing (i.e., difficulty reading and comprehending arithmetic symbols); establishing arithmetic facts (i.e., difficulty learning, automatizing and recalling arithmetic facts); and following arithmetical procedures (i.e., difficulty in calculating). The other approach suggested by McCloskey and Caramazza (1987) is to investigate the impaired information processing of arithmetic observed in children with dyscalculia that leads to various performance patterns such as difficulties in comprehending as opposed to expressing numerical information, processing numbers written in numerals rather than in words, understanding individual digits in written numbers as opposed to the place of each digit, and handing spoken as opposed to written information demands.

Whichever approach is used to define dyscalculia, Chia and Yang (2009) argue that the level of mathematical ability of an individual with dyscalculia falls below that expected for his/her age and intelligence. In addition, the individual also shows poor ability to conceptualize, comprehend and manipulate, i.e., to count, select and/or subitize (i.e., ability to say how many objects shown on a page without counting them) numbers, symbols and mathematical concepts, as well as problems in understanding and remembering fundamental quantitative concepts, rules, formulas and equations. Difficulties in performing mathematical operations in the correct sequence as well as solving word problems are also observed in such an individual.

Chia and Yang (2009) have defined dyscalculia as the disorder of mathematical abilities "to compute, where the level of mathematical ability falls below that expected for an individual's age and intelligence ... a syndrome that covers a wide range of life-long learning difficulties of developmental, acquired, or psychosociogenic origin with a varying degree of severity involving many aspects of mathematics in the process of learning" (p.3-4). However, "[T]he complexity of numerical processing has made defining what it means to have a specific mathematical learning disability difficult" (Butterworth, 2003, p.1).

Factors that affect Mathematics Learning

According to several studies (e.g., Chia & Kho, 2011; Geary et al., 2007), there are several important factors that affect mathematics learning: short-term memory for computation, long-term memory for storing and/or recalling mathematical information, number sense, ability to follow directions, visual-spatial perceptual abilities, speed of mathematical performance, reading skills, organizational skills, and checking for answers. Deficits in any of these factors will cause impairment in mathematics learning. Geary (1993) has identified the following three subtypes of dyscalculia: procedural difficulties (e.g., using developmentally immature strategies to solve problems); semantic difficulties (e.g., difficulty learning and retrieving mathematical facts); and visual-spatial difficulties (e.g., difficulty with the spatial representation of numbers in alignment or reversals).

Misconceptions and error patterns are often manifested when children over-generalize, i.e., jumping into a quick conclusion, or over-specialize, i.e., being too restrictive (Chia & Ng, 2010a).

Among all the learning difficulties in mathematics, the most prevalent difficulty concerns problems in storing and retrieving basic mathematical facts (Geary, 1993, 2007). In one unpublished study, Chia and Ng (2010b) found that this has to do with weak short-term memory needed for computation and solving story problems as well as poor long-term memory for mathematical information. The participating subjects in the study were found to perform poorly on the Arithmetic, Digit Span and Letter-Number Sequencing subtests of the Working Memory Index in the Wechsler Intelligence Scale for Children-Fourth Edition (WISC-IV) (Wechsler, 2003). In other words, for an accurate representation of the mathematical fact to be stored and retrieved later, a learner must hold all elements of the fact in his/her working memory simultaneously (Geary, 2007).

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Fuchs et al. (2008) and Geary (2007) have identified several cognitive correlates that are shown to affect basic mathematical performance that involves memory, attention-concentration span, processing speed, and language proficiency. Moreover, findings from additional studies (e.g., Chia & Ng, 2010b; Hecht et al., 2001) have found that measures of processing speed are good predictors of competence in mathematical computation. Other studies done by Bryant et al. (2000), Jordan and Hanich (2000), and Chia and Ng (2010a) have highlighted another equally serious problem for children struggling with basic mathematical computation or counting, is their difficulty in completing arithmetic problems that involve multiple steps.

One important area in mathematics learning concerns mathematical comprehension that plays a role in solving problem stories. Mathematical comprehension, according to Chia and Ng (2010a), consists of three components: The first being numerical knowledge, which includes representation and identification of numbers by respective written symbols, constant mathematical proportions and arithmetical symbols. Any child with difficulty in this skill may count well but unable to read numbers. Children with difficulty in this area may be slow in working out what such a sign means when they see it written down. Next is numerical order that children must be able to establish in ascending and descending orders. Any child with difficulty in this skill may find learning multiplication tables tough and tedious. The third component is verbal mathematical expression, which refers to the ability to express mathematical terms or concepts in words.

Mathematical comprehension also includes understanding words, phrases and jargons, besides the symbols, used in mathematics learning that constitute mathematical vocabulary. Bryant et al. (2008) have found in their study that limited knowledge of mathematical vocabulary led to poor mathematical comprehension and that, in turn, affected story problem solving skills. This means that inadequate vocabulary in mathematics learning can result in poor or weak performance in solving routine as well as non-routine problem stories, especially when a child does not know what the problem story is all about, the key clues the child is to look out for, and what he/she is supposed to solve (Ng, 2005).

Moreover, mathematical comprehension includes background information and daily life experiences as well as analytical skills needed for comprehending the story problem(s) as well as looking for key clues required to solve the problem(s). It also involves more than mathematical vocabulary. It precludes mathematical sense (logic) as in the following illustration: A=C, B=C, and A=B. Logically speaking, A=B since both share the same answer C. Mathematical comprehension is conceptually dense and difficult; unlike reading, contextual clues are limited or even non-existent for many story problems (Bryant et al. 2000; Wendling& Mather, 2009).

There are sporadic reports on individuals who can perform lightning computation but whose mathematical comprehension is so severely impaired that they are unable to solve any mathematical problem story. Such individuals, who can be of normal intelligence or are mentally challenged, have been described as having hypercalculia (Gonzalez-Garrido et al., 2002) or savant syndrome (Chia, 2008). Often individuals with hypercalculia are either autistic savants (Chia, 2008) or autistic crypto-savants (Rimland, 1990).

Finally, the attitude toward mathematics learning can also impact an individual's performance. According to Montague (1996), "[A] history of academic failure can inhibit the student's desire to perform in mathematics as well as negatively impact his or her self-confidence regarding mathematics" (p.85). Hence, such "early failures in mathematics

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learning can result in anxiety about performance in mathematics learning and this can continue into high school, college and adulthood" (Wendling& Mather, 2009, p.169).

Assessment of Mathematical Abilities Using TOMA-2

There are fewer diagnostic mathematics tests than diagnostic reading tests. However, according to Pierangelo and Giuliani (2009), "mathematics assessment is more clear-cut. Most diagnostic mathematics tests generally sample similar behaviors" (p.143). Many of these diagnostic mathematics tests developed for the purpose of assessing the mathematical competencies cover a wide variety of mathematical concepts of school-age children. Salvia and Ysseldyke (2007) have identified three types of classifications to be involved in diagnostic mathematics tests. Each classification measures certain mathematical abilities as given below (see Pierangelo& Giuliani, 2009, p.143): (1) Content: This consists of numeration, fractions, geometry, and algebra; (2) Operations: This consists of counting, computation, and measuring; and (3) Applications: This consists of measurement, reading graphs and tables, money and budgeting time, and problem solving.

According to Brown et al. (1994), "[A]lthough many tests have been developed to measure specific math skills, few have been designed to measure the additional factors (i.e., attitudes toward mathematics, understanding the language of mathematics, and familiarity with general mathematical information found in everyday life)" (p.1). For this reason, the Test of Mathematical Abilities-Second Edition (TOMA-2) "has been developed to provide standardized information about attitudes, vocabulary, and general cultural applications f mathematical information, as well as two major traditional skill areas – story problems and computation" (Brown et al., 1994, p.1).

The TOMA-2 was chosen as the instrument of measurement for this comparative study. The reasons are fourfold as given in TOMA-2 examiner's manual (see Brown et al., 1994, p.3): Firstly, it is used to identify children who are significantly below their peers in mathematics and who might profit from supplemental help. Secondly, it can be used to determine particular strengths and weaknesses among mathematics abilities. Thirdly, it can also be used to document progress that results from special interventions. Lastly, it provides professionals who conduct research in the area of mathematics with a technically adequate measure.

More importantly, the TOMA-2 has been empirically investigated for gender and racial bias. It was normed using a standardization sample comprising 2,082 students, ranged in age from 8-0 to 18-11 and resided in 26 states in the United States of America, between 1990 and 1992 (see Brown et al., 1994, for more detail). "The characteristics of the sample are similar to those reported in the 1990 Statistical Abstract of the United States for the population as a whole" (Pierangelo& Giuliani, 2009, p.146).

The TOMA-2 has five subtests, four in the core battery (Vocabulary, Computation, General Information, and Story Problems) and one supplemental subtest (Attitude toward Mathematics). "The results of the test may be reported in standard scores, percentiles, and grade or age equivalents. The standard scores of the core battery are combined to comprise a total score called the Mathematics Quotient (MQ)" (Pierangelo& Giuliani, 2009, p.146). All five subtests, which measure the different aspects of mathematical ability, are briefly discussed below:

Vocabulary (VO)

This subtest measures the ability to understand words used in mathematical thinking.

Computation (CO)

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This subtest measures the ability to solve an array of arithmetical problems.

General Information (GI)

This subtest measures the knowledge of mathematics as it is used in everyday situations.

Story Problems (SP)

This subtest measures the ability to read and solve written problems.

Attitude toward Mathematics (AtM)

This supplemental subtest measures an examinee's attitude toward mathematics learning.

The internal consistency reliability coefficients of the five subtests including the computation of MQ in the TOMA-2 range between 0.73 and 0.98 with an average range between 0.84 and 0.97 depending on the different age groups from 8 years-old to 18 years-old (see Brown et al., 1994, p.28, for more detail). The test-retest reliability coefficients for all five subtests including the computation of MQ average between 70 and 92 for age groups from 10 years-old to 14 years-old (see Brown et al., 1994, p.29, for more detail).

The TOMA-2 subtests were inter-correlated using the entire normative sample as subjects. The resultant coefficient are shown in Table 1, where $p < .01$ for all coefficients.

Table 1. Inter-correlation Reliability Coefficients of the TOMA-2 Subtests

Subtests	VO	CO	GI	SP
VO				
CO	0.62			
GI	0.59	0.59		
SP	0.55	0.60	0.58	
AtM	0.09	0.21	0.11	0.21

Key: VO = Vocabulary; CO = Computation; GI = General Information; SP = Story Problem; AtM = Attitude toward Mathematics

According to Chia and Kho's (2011) interpretation, the inter-correlation of the TOMA-2 subtests suggests that CO/VO and SP/CO have sufficiently reliable coefficients, while those of the GI/VO, GI/CO, SP/VO and SP/GI are low. "There is poor or no reliability for inter-correlation between the supplementary AtM subtest and each of the other four core subtests" (Chia & Kho, 2011, p.101). The inter-correlation of the TOMA-2 subtests is expressed in terms of a cognitive equation below:

Mathematics learning $\rightarrow \{AtM+GI+(CO\times VO)+(CO\times SP)\} \rightarrow$ Mathematics Quotient ... Equation 1

Mathematics learning $\rightarrow \{AtM+GI+CO(VO+SP)\} \rightarrow$ Mathematics Quotient ... Equation 2

where the + symbol suggests low or no inter-correlation between subtests and the × symbol suggests sufficient or significant inter-correlation between subtests.

The first equation shows how the four components – AtM, GI, (CO×VO) and (CO×SP) – are joined by the + symbol suggesting low or no inter-correlation between or among them. However, CO is sufficiently correlated to VO and SP in terms of (CO×VO)+(CO×SP). The second equation shortens these correlations (CO×VO)+(CO×SP) into CO(VO+SP). The two

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symbols + and × are not to be confused with or mistaken for the symbols of addition and multiplication nor do they function as such, respectively.

In the normative study of TOMA-2 carried out in 1990-1992, the standard scores made by 38 students (5% of the 2082 participants) with learning disabilities were found to be appreciably lower than normal. However, no background information (e.g., age, gender and race) about these students was provided by authors of TOMA-2.

According to Brown et al. (1994), "[T]he average standard scores earned by the children with learning disabilities were Attitude toward Math=9; Vocabulary=6; Computation=6; General Information=7; and Story Problem=7" (p.36). 10 is the score expected of typical students and any score lower than that indicates that this group of students evidences learning challenges, especially problems involving performance in mathematics. This conclusion has been firmly supported by the unusually low MQ of 79 that was also observed for this group (Brown et al, 1994). As such, Chia and Kho (2011) have termed this TOMA-2 based Dyscalculia profile the 6-6-7-79-79 (based on the standard scores of the VO-CO-GI-SP-AtM-MQ).

Comparison of Published TOMA-2 Based Studies on Singaporean Children with Mathematics Learning Disabilities

A literature search for TOMA-2 based studies in Singapore at the National Institute of Education Library has yielded only two papers. Both papers reported findings done by Chia and his co-authors. In the first paper (i.e., Chia & Ng, 2010b), Chia and Ng (2010a, 2010b) did two separate studies on the performance of Singaporean students with dyscalculia in mathematics learning.

In their first study (see Chia & Ng, 2010a, for more detail), error patterns in computation of whole numbers were carefully investigated to determine the type of profile expected of a student with dyscalculia. It was difficult to identify a definite profile that categorized all students with dyscalculia basing on computation error patterns alone. In their second study (see Chia & Ng, 2010b), using the same cohort of participants in their first study, the TOMA-2 was administered in search of the Dyscalculia profile as defined by 6-6-7-7-9-79, which has been mentioned earlier. The findings failed to reveal or identify those who could fit perfectly into the 6-6-7-7-9-79 profile. Based on the mean standard scores for all the subtests and MQ, the Dyscalculia profile from Chia & Ng's (2010b) study was 7-9-6-8-8-87 and it differed from TOMA-2 profile of 6-6-7-7-9-79.

In the second paper, Chia et al. (2011) did another study on Singaporean students, ranged in age between 9-1 and 9-11, with selective mutism (SM) and dyscalculia (or simply abbreviated into SM+DYSC) in search of a cognitive equation for mathematics learning of learners who performed poorly in mathematics. The results showed the Dyscalculia profile in this cohort of SM+DYSC students was 6-8-7-6-8-79. Except for CO subtest, the other standard scores fell below or at the exact average scores of the TOMA-2 based profile for dyscalculia.

Comparison of TOMA-2 Results between American and Singaporean Students with Dyscalculia

The aim of this paper is to compare the standard scores of the subtests of the TOMA-2 scored by 38 American and 40 Singaporean students with dyscalculia that have been previously published.

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Table 2. Mean Standard Scores of the TOMA-2 Subtests (N = 40)

Subtests	VO	CO	GI	SP	AtM	MQ
Mean	8.5	9.6	9.6	6.5	7.0	90
Standard Deviation	1.68	3.08	2.34	1.59	1.17	9.72

Key: VO = Vocabulary CO = Computation
GI = General Information SP = Story Problem
AtM = Attitude toward Mathematics

The author has selected results (see Table 2) of the study done by Chia and Kho (2011). In that study, students had been failing in their mathematics class tests and school examinations since Primary 3. The TOMA-2 was administered to determine which areas of mathematics learning were lacking.

I. The mean VO standard score was 8.5 (SD=1.68). 18 participants (45%) had VO standard scores above the mean. 22 (55%) failed to meet this criterion.

II. The mean CO standard score was 9.6 (SD=3.08). 20 participants (50%) had CO standard scores above the mean and an equal number of them failed to meet it.

III. The mean GI standard score was 9.6 (SD=2.34). 18 participants (45%) had GI standard scores above the mean and the remaining 22 of them (55%) failed to meet it.

IV. The mean SP standard score was 6.5 (SD=1.59). 22 participants (55%) had SP standard scores above the mean, while 18 of them (45%) failed to meet it.

V. The mean AtM standard score was 7.0 (SD=1.17). 28 participants (70%) had AtM standard scores above the mean with only 12 (30%) failing to meet it.

VI. The mean MQ was 90 (SD=9.72). 25 participants (62.5%) had mean MQ at 90 or above while 15 of them (37.5%) failed to meet it.

From Chia and Kho's (2011) study, SP and AtM were noted to be the lowest in their respective mean standard scores in the low average range. CO and GI showed the best results followed by VO, and all three mean standard scores were in the average range. In other words, the majority of the participants in the study performed poorly in terms of their attitude toward mathematics learning and their performance in solving story problems in mathematics.

Table 3. Inter-correlation Reliability Coefficients of TOMA-2 Subtests (Chia & Kho, 2011)

Subtests	VO	CO	GI	SP
VO				
CO	0.19			
GI	0.27	0.63		
SP	0.61	0.27	0.25	
AtM	0.24	0.75	0.63	0.51

Key: VO = Vocabulary CO = Computation
GI = General Information SP = Story Problem
AtM = Attitude toward Mathematics

Table 3 shows the inter-correlation reliability coefficients between and among the four core subtests (VO, CO, GI and SP) and one supplementary subtest (AtM) of the TOMA-2, and the mathematics quotient (MQ) computed from the standard scores of the four core subtests.

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Among the subtests, the lowest inter-correlation reliability coefficient was between CO and VO subtests with a correlation coefficient *r* of .19. This means there was no good correlation between the computation (CO) and mathematical vocabulary (VO). This finding is a big contrast to that of *r*=.62 between CO and VO reported in the TOMA-2 manual. The inter-correlation reliability coefficient *r* between AtM and VO was .24 (in TOMA-2, *r* for AtM/CO is .09, which is even lower) while that between SP and GI was .25 (in TOMA-2, the *r* for SP/GI is .58, which is higher), and followed by two other poor inter-correlation reliability coefficients *r*'s of .27 between GI and VO as well as SP and CO (in TOMA-2, the *r* for GI/VO is .59 while the *r* for SP/CO is .60; both *r*'s are higher).

In other words, findings from Chia and Kho's (2011) study suggested that there was hardly any reliable correlation between AtM and VO, and also between SP and GI. The same could also be explained for that between GI and VO, and between SP and CO. Chia and Kho (2011) used the symbol + to represent poor or no correlation between and among the TOMA-2 subtests as shown here: CO+VO; AtM+VO; SP+GI; GI+VO; and SP+CO.

The correlation reliability coefficient *r* between AtM and SP was .51 and was considered low or poor. According to Chia and Kho (2011), this finding of their study suggested the low or poor impact of the general attitude of the Singaporean students with dyscalculia toward AtM on their performance in solving SP. The same finding was also noted in the correlation reliability coefficient *r*=.55 between MQ and VO (see Table 4). In other words, the finding suggested that VO subtest was a poor indicator/predictor of MQ. The symbol + was used by Chia and Kho (2011) to represent low or poor correlation between the subtests as well as between a subtest and MQ.

Table 4. Inter-correlation Reliability Coefficients between TOMA-2 Subtests and MQ (Chia & Kho, 2011)

Subtests	VO	CO	GI	SP	AtM
MQ	0.55	0.84	0.70	0.67	0.83

Key: VO = Vocabulary; CO = Computation; GI = General Information; SP = Story Problem; AtM = Attitude toward Mathematics; MQ = Mathematics Quotient

However, there were acceptable correlation reliability coefficients *r*'s between SP and VO (*r*=.61), GI and CO (*r*=.63), AtM and GI (*r*=.63), MQ and SP (*r*=.67), MQ and GI (*r*=.70), and AtM and CO (*r*=.75). The only sufficiently reliable correlation coefficients *r*'s were the ones between MQ and AtM (*r*=.83), and between MQ and CO (*r*=.84) (see Table 4). These findings from Chia and Kho's (2011) study suggested that AtM and CO subtests were good indicators/predictors of MQ. Chia and Kho (2011) used the symbol × to represent adequate or reliable correlation between and among the TOMA-2 subtests and between each subtest and MQ.

At the heart of mathematics learning is CO and SP, i.e., mathematics learning → CO+SP. As a result of Chia and Kho's (2011) findings, the following cognitive equation for mathematics learning has been created:

Mathematics learning →{AtM[GI(CO)]+VO(SP)}→ Mathematics Quotient ... Chia & Kho (2011)

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This cognitive equation for mathematics learning is completely different from the one Chia and Kho (2011) have formulated basing on the inter-correlation reliability coefficients of TOMA-2 subtests and MQ as shown here:

Mathematics learning → {AtM+GI+CO(VO+SP)}→ Mathematics Quotient ... TOMA-2 manual

There are many possible explanations why the cognitive equations for mathematics learning are different. The first possible explanation that Chia and Kho (2011) have argued is that the cohort of Singaporean students with MLD is totally different from the 38 American students identified in the TOMA-2 normative study (1990-1992) in terms of the sample composition and sample size. The sample used in the study done by Chia and Kho (2011) is rather small (N=40) and all the participants (23 Chinese, 12 Malays and 5 Indians) in their study came mainly from neighborhood schools in the western region of Singapore. Unlike the Singaporean sample, the TOMA-2 normative study (1990-1992) used a huge sample size of 2082 American students residing in 26 states, but only 38 of them were identified to have MLD without providing other detailed background information (e.g., age, gender and race) about them. The author is aware that without the essential background information of these 38 American students, it would not be a fair comparative study between the two samples of students with dyscalculia.

In the second possible explanation, Chia and Kho (2011) have pointed out that the way mathematics is taught in Singapore is certainly very different from how it is done in the United States. This is more likely because of the different mathematics curricula used and/or the scope and sequence of the topics covered in mathematics taught in the American and Singapore schools.

A third explanation is our need to consider failure in mathematics learning as a multi-faceted phenomenon. The possible causes are many and it is difficult to pinpoint any specific cause that results in mathematics learning failure. In fact, failure in any of the TOMA-2 subtests is not an adequate explanation for poor performance in mathematics. Chia and Kho (2011) have argued that "[T]he sum of all the four core subtests and one supplementary subtest in terms of their respective standard scores does not equal to mathematics learning in its entirety. There are many other factors (e.g., processing speed and attention span) not included or measured by the standardized test" (p.113).

A fourth explanation is that it would not be fair or accurate to compare the results of the two TOMA-2 based studies because there is a wide chronological gap. The TOMA-2 normative study was done between 1990 and 1992, while Chia and Kho's study was done in 2011. The reason is that the two samples of participating students were from the different chrono-systems, which refer to the dynamic, ever-changing nature of the individual learner's environment and it includes the patterning of events and transitions over the lifespan as well as socio-historical circumstances (Bronfenbrenner, 1979). For example, education policy is one transition.

Each time a new Education Minister is appointed, new policies such as pedagogical issues related to mathematics education are introduced and implemented in schools. As a result, both teachers and students will be affected. As an example of socio-historical circumstances, consider the way mathematics is taught in 2000's and how it was done back in the 1990's.

A fifth explanation is that AtM subtest should not be included in the cognitive equation for mathematics learning. There are two reasons why AtM should be excluded from the equation. Firstly, in the TOMA-2, AtM subtest is a supplementary, not a core, test, and is not used in the computation of MQ. Secondly, AtM has more to do with a learner's affect rather than the cognitive process. However, this is not say that AtM has no impact on the cognitive process of mathematics learning. In fact, this highlights the complexity of mathematics learning that involves more than just its cognitive process.

This suggests a need to examine also the following learning blocks or factors that will impact on mathematics learning: (1) the innate abilities that come along with the capability to learn and acquire mathematical skills and abilities; (2) the sensory-perceptual-motor skills and abilities; (3) the adaptive behavioral skills and abilities; (4) the social-emotional/affective behavioral skills and abilities; (5) the cognitive skills and abilities; and (6) instructional skills and abilities (see Chia, 2011, for more detail). In other words, the cognitive equation for mathematics learning forms only a small part or constitutes one of the many parts of a bigger multi-faceted equation that we should be looking at.

CONCLUSION

The current cognitive equation for mathematics learning involving the TOMA-2 subtests is inadequate to explain the process in its entirety. There are other essential but intangible learning blocks such as mathematical sense and mathematical comprehension that are missing from the equation. As explained earlier, the mathematics learning as a process is more than the sum of the five TOMA-2 subtests put together. According to Chia and Kho (2011), "[T]he subtests are only useful in measuring certain sub-components (e.g., VO and GI are parts of the bigger block called mathematical comprehension) or sub-processes (e.g., CO and SP) of mathematics learning" (p.114).

Mathematics learning is not a simple process consisting only computation and story problem solving. Computation and story problem solving are like decoding and encoding in the reading process. First, all students need to have numerical knowledge awareness (number sense) like what phonological knowledge awareness (phonemic sense) is to a reader. Next, in computation, students also must be able to use arithmetic operations to work out their answers in form of mathematical expressions. Through the process of computation, students will gain mathematical sense in the same way like reading sense helps to establish meaning of what is read. This mathematical sense is called logic. In addition to computation and logic, story problem solving ability is a must to establish mathematical comprehension, which involves analytical skills in the same way a reader uses the skills of discourse analysis to critique what he has understood from the reading.

To sum up the essential components involved in the mathematical learning, the following model of mathematics learning process has been proposed (see Figure 1):

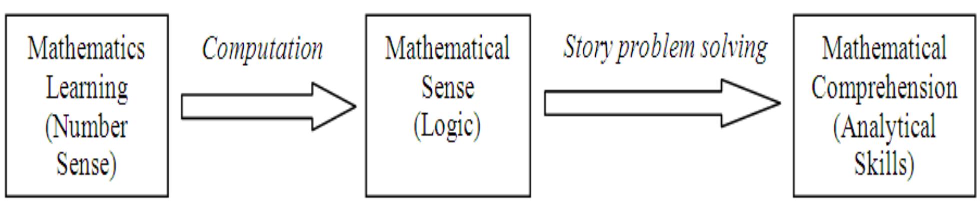

Figure 1. The Proposed Model of Mathematics Learning Process

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