COMMON CONCEPTS OF OPERATIONS OF ADDITION AND SUBTRACTION ON WHOLE NUMBERS AND FRACTIONS

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ABSTRACT

This paper discusses a short research notes of operations of addition and subtraction on whole numbers and fractions. The notes are about intertwining of operations on whole numbers and fractions in the frame of realistic mathematics education. In conventional approaches, the operations of addition and subtraction on whole numbers and fractions look like two completely separated entities. . They are also, currently, taught at school as two different topics that apparently have no connection each another. On the other hand, naturally, if we investigate further we will find that the two entities intertwine closely.

Keywords: whole number, fraction. intertwine, conventional, and realistic.

INTRODUCTION

Mathematics currently at almost all levels of schools in Indonesia is considered as a burden by majority of students, and even this negative image affects people who do not directly involved in the teaching and learning process such as parents and society (Makkink, 2010). What they know is mathematics is a boring, difficult, and confusing subject (Sembiring, 2010). One of the factors that might contribute to minimize this unwanted images and responses is an introduction to a friendly and realistic instructional approach that enables students to see mathematics as a tool to solve problem rather than mathematics as a problem for them (Widjaja et, al, 2010).

Contrary to the expectation, the instructional approach performed in most mathematics classroom today stress on the mechanistic, procedural, and meaningless (Zulkifli & Mohini Mohamed, 2007). The condition will get worse when teachers simply teach it by following page by page of mathematics textbooks. This research notes highlight common characteristics of operations on whole numbers and fractions based on Realistic Mathematics Education (RME) and this characteristic cannot be seen by those who only memorise standard procedure and apply it. It is expected to bring some benefits to teachers and students to see mathematics in a meaningful way.

BACKGROUND

Mathematics teaching and learning, in this case the teaching on operations of addition and subtraction on whole numbers and fractions, currently look like two separate entities that have a complete difference, operations on whole numbers in one side and operations on fractions on the other side. There are specific rules for the operations on each number. The operations of addition and subtraction on whole numbers can be performed directly without any other considerations, and the same operations on fractions must conform to the standard procedure concerning the numerator and denominator.

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The two entities were studied without any emerged questions. Students just memorize the rules and procedures and retrieve them when they were needed. On the other hand, (Freudenthal, 1991) suggested that in order for students to see mathematics as a realistic entity, they should see the interrelationship between strands (topics), in this case the relationship between operation of addition and subtraction on whole numbers and fractions. This suggestion is concerning with a characteristic of RME (the last characteristic of the five characteristics) which encourage to look at the relationship between topics (intertwining of various learning strands).

Realistic Mathematics Education is an approach in mathematics teaching and learning which focus on the meaningfulness of the mathematics contents through the students' totally involvement in the learnt material, experiencing to learn and work on mathematics, and at the end the students will create mathematics for themselves (Freudenthal, 1991). The approach is the opposite of the traditional approach which focus on the ready-made mathematics to be learnt through memorizing and retrieving the facts, symbols, procedures, etc.

The implementation of RME in classroom will affect the ways the teachers teach mathematics. In RME classrooms, the role of the teachers shift from traditional story teller to the comfortable environment provider that enables students to reinvent mathematics on their own and facilitate students' learning process.

Freudenthal (1991) classified mathematics education into four types of mathematization, they are:

1. Mechanistic, it is a well-known as traditional approach. The process of mathematics instruction is emphasized on drills and patterns. Students have no opportunities to think why the patterns and process work in that way. Students are treated like machines or computers to do as what they are instructed to do.
2. Empiristic, this approach in emphasis on horizontal mathematization and less focus on vertical mathematization.
3. Structuralist, this approach emphasis on vertical mathematization which ignore horizontal mathematization. This approach what the "new math" movement implement and practice. The movement use set theory as the basis to build mathematics concept.
4. Realistic, the realistic approach treat equally between horizontal and vertical.

The clearer illustration (the summary) of this approach can be seen in the table below.

Table 1. Type of Mathematics Education (Freudenthal, 1991)

Type	Horizontal	Vertical
Mechanistic	-	-
Empiristic	+	-
Structuralist	-	+
Realistic	+	+

From the table above, the realistic approach performs both horizontal and vertical mathematization. In the process of teaching and learning, RME appear in five characteristics:

1. Contextual problems as starting points to explore phenomenology;

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2. Using model to bridge between informal and formal mathematics;
3. Students contributions;
4. Interactivity;
5. Intertwining of various learning strands.

The RME is an approach in mathematics teaching and learning that make mathematics more visible, meaningful, and interconnected between topics and strands. There is a relatively new topic in mathematics education (number sense) that in line with the characteristic to RME regarding to its substance. Due to its important, number sense has received a great attention among mathematics educators and mathematics education researchers (Anghileri, 2000; Yang & Wu, 2010).

Number sense is a general understanding about numbers and operations as well as its application into computational settings (Yang, 2005). Understanding numbers and operation flexibly will enable students their subject beyond a set of meaningless symbols and formulas.

If we look further, we will find that number sense and RME have a common characteristic, particularly in term of the meaningfulness of mathematics learning. Number sense can be seen as a product of RME approach which stress on the understandable learning material.

The idea of the intertwining operations between whole numbers and fractions emerged in an enactment of design research, the underpinning research of RME, which focuses on number sense. The class tried to see the topic at different viewpoints from the routine activities. The question of "Is there any relationship between operations on whole numbers and operations on fractions?" leads to the further investigation.

This paper explicates on the fifths characteristic (intertwining of various learning strands) of RME. The learning strands we will elaborate is the relationship between operation of addition and subtraction on whole numbers and fractions. The following diagram shows the interconnected between number sense and RME.

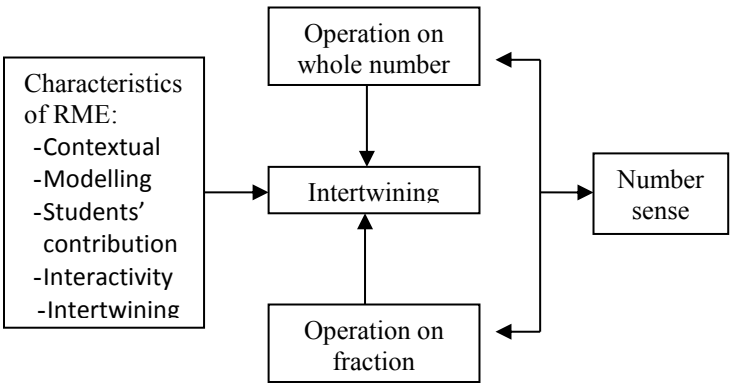

Figure 1. Intertwining of operations and number sense

ILLUSTRATION OF CONVENTIONAL AND REALISTIC APPROACH IN OPERATION ON WHOLE NUMBERS AND FRACTIONS

Conventional Approach

Conventional approaches in mathematics teaching and learning use ready-made formulas to be applied in classrooms. It can be said as a top-down approach in which students just apply the formulas prescribed by experts without knowing where they come from and why they work in such that ways.

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Teaching addition and subtraction on whole numbers in conventional approach is simply adding or subtracting the numbers. The activities are then followed by a set of routine problems to drill students on the topic. Addition and subtraction on fractions will be little complicated rules and procedures that must be followed. These rules that make most students feel anxious because it is hard for them to follow, remember, and apply in the exercise. The addition and subtraction on whole numbers and fractions look like the two entities that have no connection to each others. Below are the sample of operations from conventional textbook on whole numbers and fractions.

Whole numbers

(i) 7 + 3 = ...
(ii) 23 + 14 = ...
(iii) 8 – 3 = ...

Fractions

(i) $\frac{2}{5}+\frac{3}{5}=\cdots$
(ii) $\frac{1}{2}+\frac{1}{3}=\cdots$
(iii) $\frac{1}{3}-\frac{7}{8}=\cdots$

For the whole numbers, the current common practice on such the operation is that the numbers can be directly executed without any other considerations. The result of the problems respectively are 10, 37, and 5.

For the fractions, the operations are performed with the conditions of common denominators. The first problem can be executed directly by simply adding the numerators (the result is $\frac{5}{5}$). The second and the third problems need to change to satisfy the conditions. The second problem need to adjust the denominator to the least common multiple of denominator 2 and 3. The problem will be changed to be $\frac{3}{6}+\frac{2}{6}$ so the result will be $\frac{5}{6}$. The same way is also work for the third problem ($\frac{1}{3}-\frac{7}{8}$), but the only difference and the only difference is at the process of subtraction.

If we look at the operations on these two kinds of numbers (operation on whole numbers and fractions above), the impression that come to students' mind is they are quite different. This believe make students learn that nothing to consider when working with the whole numbers, except simply adding or subtracting. On the other hand, it needs an extra work in adjusting the denominator to execute the operations on fractions.

Realistic Mathematics Education Approach

Contrary to conventional approach, these research notes which are based on RME introduce an approach that will enable students to see the mathematics in a more meaningful way and holistic conceptual approaches. In relation to addition and subtraction of whole numbers and fractions, to make the topic make full of sense, one of the principles of RME should be introduced here (the principle of intertwining).

If we look deeper on the subtraction and addition on whole numbers, we will find that something is understood and not being written explicitly, but it is exist implicitly. If we look under the veneer of routine procedure we will notice that whole numbers also need objects with the same characters to be added or subtracted. For example, the aforementioned addition 7 + 3 carry meaning that this addition is to combine two sets of the same character objects,

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such as 7 chairs and 3 chairs or 7 fruits and 3 fruits. It will not work if we add 7 chairs and 3 fruits. In the case of 7 fruits + 2 fruits, fruit is equivalent to denominator of fraction.

This case can be compared to a case in linguistics. In linguistics, a sentence must have subject and verb (Demirezen, 2012). Sometimes, the condition of a command sentences with there is no explicitly written and spoken subject. The command sentence likes "Go!" look like that there is no subject available. That is true, but if we look deeper, we will find that the subject is understood even though it is not spoken or written (Pyle & Munoz, 1995). The subject of the sentence could be you or you all. It is impossible that the command "Go!" is instructed without anyone who will do it.

In relation to the command sentence in linguistics, the addition 7 + 3 must have the same characteristics of objects. For example, 7 cows + 3 buffalos cannot be executed spontaneously, but it has a way to see the common character that the cow and buffalo have. It will be easier when the addition is shifted to the addition 7 cows + 3 buffalos to 7 animals + 3 animals and the result is 10 animals.

If we look at first fraction above ($\frac{2}{5}+\frac{3}{5}$), the problem can be seen as equivalent as the problem of addition 2 books + 3 books which has the result 5/5. In the same way of addition, $\frac{2}{5}+\frac{3}{5}$ (2 fifths + 3 fifths) is equal to $\frac{5}{5}$ (5 fifths). The additional works because of $\frac{2}{5}$ and $\frac{3}{5}$ have the same denominators. The following are some illustrations of how they, the two kind of addition problems, look like in the concrete view.

2 books + 3 books

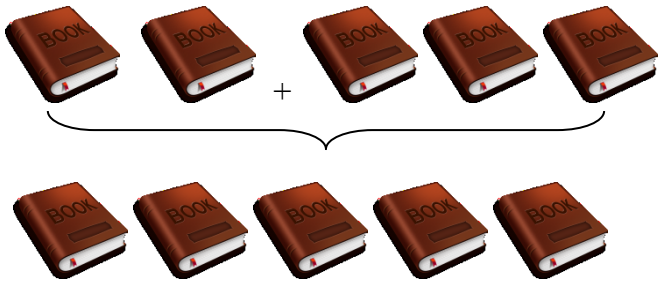
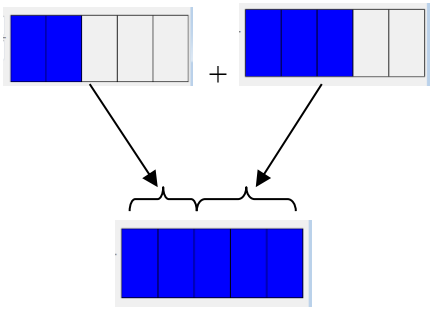


Figure 2. Example whole numbers addition in concrete view

$\frac{2}{5}+\frac{3}{5}$ (2 fifths + 3 fifths)



Figue 3. Example of fractions addition in concrete view

In other case, the addition of whole numbers 2 + 3 cannot be executed if the objects are not the same. For example, 2 books + 3 pencils cannot be executed without finding the same characteristic of the objects. Such addition may be converted to 2 writing tools + 3 writing toos to performed the addition with the result 5 writing tools. This way of thinking is also suitable for fraction addition. The addition $\frac{1}{2}+\frac{1}{3}$ cannot be executed before finding the same

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chracteristics (common denominator). The addition must be converted to $\frac{3}{6}+\frac{2}{6}$ and then the result will be obtained.

CONCLUSIONS

As an utmost important subject, mathematics should have a good and interesting images among students. This is important because it is highly correlated to the achievement of the subject. To achieve the goal, particularly at prmary schools, mathematics teaching learning should be perfomed as conccrete as possible and abstraction should be at a lowest level. In other words, the abstract concept should be postponed until students undertsnd it in concrete side.

RME has given an alternative solution to mathematics to be a concrete subject to students' mind. This is one of the effort to make mathematics more familiar, interesting, and meaningful. In the case of numbers and operations and in line with RME, number sense give much contribution to make numbers and operation look concrete, meaningful, and understandable.

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